Lecture IV
Confidence intervals
Hypothesis tests

STA9750

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Baruch College CUNY

Logistics

- HW1 up on the website, ***due 09/26***
- New class dynamic:
  - I'll upload a handout and datasets at least 24h ***before*** lecture with SAS commands
  - I'll expect you to have looked at the handout and tried things out
  - In lecture, I'll go through the handout and answer questions you had as you went through it
  - After that, I'll lecture about some new topic, which will be the object of the new handout

Today

- Confidence intervals
- Hypothesis testing
- Review of last week’s handout

Confidence intervals

- **Want:** estimate some unknown θ
- **Goal:** Provide a range of values that seem "good," not just one point estimate…
- Suppose that the data X are independent and identically distributed (iid)

$$x_i \overset{\text{iid}}{\sim} P_\theta, \ \theta \text{ unknown}$$

- A *random* interval $[L(X), U(X)]$ is a $(1-\alpha)\cdot 100\%$ confidence interval (CI) for θ if, for all possible θ:

$$\boxed{P(L(X) \leq \theta \leq U(X)) \geq 1-\alpha}$$

CIs… Interpretation?

- If you use 95% CIs all your life, roughly 95% (or more) will catch the "true value" [if the assumptions under which you derived the CI sort of hold…]
- **What about this particular one?** The 95% guarantee refers to repeated sampling, not a particular dataset

https://istats.shinyapps.io/ExploreCoverage/

Examples

- Normal mean: $x_i \stackrel{\text{iid}}{\sim} N(\mu, \sigma^2)$, then $(1-\alpha)\%$ CI for μ

n sample size

$\bar{x}$ sample mean

s sample standard deviation

$t_{n-1,\alpha/2}$ quantile of Student-*t*, n-1 deg. of freedom

$$C_\alpha = \overline{x} \pm t_{n-1,\alpha/2} \frac{s}{\sqrt{n}}$$

It's "exact": $P(\mu \in C_\alpha) = 1 - \alpha$

If you need more review, this might be helpful:
http://www2.stat.duke.edu/~vp58/sta111/lecture13.pdf

Hypothesis testing

- Want to choose between competing hypotheses H_0 (null) and H_1 (alternative)

Our decision

Truth		Don't Reject H_0	Reject H_0
	H_0 is true	OK	Type I error (false positive)
	H_1 is true	Type II error (false negative)	OK

Common practice

1. Assume that H_0 is true
2. Find the *p*-value: probability of finding data as "extreme" or "more extreme" (in a direction favorable to H_1) than the observed data under H_0
3. Reject H_0 if the *p*-value is below a pre-specified threshold α. Otherwise, don't reject H_0

If you follow this procedure, you will wrongly reject the null $\alpha \cdot 100\%$ of the time

CIs and hypothesis tests

- Given a $(1-\alpha)\cdot 100\%$ CI, you can do an α-level hypothesis test for $H_0:\theta=\theta_0$ against $H_1:\theta\neq\theta_0$ by checking whether the interval contains θ_0 or not

- Similarly, given a test, we can find CIs by finding the values of θ_0 for which $H_0:\theta=\theta_0$ isn't rejected

Two-sample tests, normal mean

$$y_{i1} \overset{\text{iid}}{\sim} N(\mu_1, \sigma^2) \qquad H_0 : \mu_1 = \mu_2$$
$$y_{l2} \overset{\text{iid}}{\sim} N(\mu_2, \sigma^2) \qquad H_1 : \mu_1 \neq \mu_2$$

- If $i \neq l$, y_{i1} and y_{l2} are independent
- **Independent samples:** also independent if $i = l$
- **Paired tests:** dependent if $i = l$
- Usually, paired observations are 2 measurements on the same experimental unit
  - *Example:* measurements before and after some treatment

Contingency tables

- We have two categorical variables
- We want to know if the distribution of one of the variables depends upon the levels of another
- *Example*:
  - Variables: Policy support and party affiliation
  - Does policy support depend on party affiliation?
- <u>Chi-squared test</u>: H_0: variables are independent
  H_1: variables are dependent

ANOVA

- Testing whether all the means from different groups are equal
- We have normal data coming from *k* groups

$$y_{ij} \overset{\text{ind}}{\sim} N(\mu_j, \sigma^2)$$

$j \in \{1, 2, \ldots, k\}$ group

$i \in \{1, 2, \ldots, n_j\}$ observation within group *j*

$$H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$$

$$H_1 : \text{at least 2 pop. means are different}$$

Now... Go through handout

- Find *p*-values and CIs with SAS